Abstract

In this thesis, the problem of scheduling n jobs on a single machine is considered to minimize Multiple Objective Function (MOF). There are two aims for this study, the first aim is to find the optimal solution for the number of tardy jobs, maximum tardiness and makespan with unequal release dates, no preemption is allowed, this problem denoted by $1/r_j/\sum_{j=1}^{n} U_j + T_{max} + C_{max}$, to the best of our knowledge this problem is not studied before. The second aim is to find the best possible solution for a function of two and three criteria $\sum_{j=1}^{n} U_j, T_{max}$ and C_{max} in hierarchical (or lexicographical), optimization method, with the jobs are available at zero time and no preemption is allowed.

For the first aim, ten special cases are derived and proved that yield optimal solutions. Also, Branch and Bound (BAB) algorithm is proposed with two lower bounds (LB_1, LB_2) and two upper bounds (UB_1, UB_2) that introduced in this thesis, in order to find the exact (optimal) solution. Two dominance rules are suggested and proved which help in reducing the number of branches in the search tree. Results of extensive computational tests show the proposed (BAB) algorithm is effective in solving problems up to (40) jobs at a time less than or equal to (30) minutes. In general, this problem is strongly $\mathcal{NP}$-hard.

For the second aim, three algorithms are proposed to find the best possible solution for each of the following three problems (P_1, P_2 and P_3). Two of these algorithms solve the problems (P_1 and P_2). The third algorithm is the Branch and Bound (BAB) is used to find exact (optimal) solution for the problem $1//\sum_{j=1}^{n} U_j + T_{max}$. In this thesis the problems P_1 and P_2 proved are Open and the problem P_3 proved is $\mathcal{NP}$-hard. An equivalence for some problems are proved which reduces the algorithms of solutions from twelve algorithms to three algorithms only.